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A micro-mechanical model for crack growth resistance of particulate-reinforced metal-matrix composites

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Particulate-reinforced metal-matrix composites are being developed to satisfy the need for high stiffness, high strength materials by combining the beneficial properties of both metals and ceramics. The use of such composites as structural members necessitates an understanding of their damage tolerance. A model relating the crack growth resistance to initiation toughness and tensile properties is developed in this paper. The crack growth toughness has also been related to fundamental microstructural properties of these composites. A critical ratio of Young's modulus to flow stress to enable stable crack growth has been established. Suitable suggestions for tailoring the microstructure to optimize crack growth toughness have been discussed.

1. Introduction

Particulate-reinforced metal-matrix composites (MMCs) comprise a class of new-generation materials whose properties can be tailor-made to suit a particular application. These composites not only possess high specific strengths and moduli at room and elevated temperatures, but also have excellent wear resistance, high thermal conductivity, low coefficient of thermal expansion and good dimensional stability. Particulate-reinforced metal-matrix composites also exhibit isotropy, unlike continuous fibre-reinforced composites, and are much easier and less expensive to fabricate. The potential applications for such composites are primarily concentrated into three market sectors, namely aircraft/space/defence, automotive and sporting goods. Some typical examples are missile fins, inertial guidance control components, precision laser mirror substrates, pistons in diesel engines, brake callipers, bicycle frames, tennis rackets, etc.

Aluminium and its alloys are by far the most popular choice for the matrix. This is because of their low density, ease of processing and excellent property improvement with reinforcement. Silicon carbide alumina and boron carbide are the commonly used particulate reinforcements with silicon carbide being the most popular because of its good compatibility, high modulus, ready availability and low cost. Particulate-reinforced MMCs can be produced by either the powder metallurgy (P/M) or the liquid metallurgy route. However, superior properties are obtained for composites made by the P/M route as compared to the liquid metallurgy route, albeit at a higher cost.

The use of particulate-reinforced composites as structural materials will, however, depend to a large extent on their degree of damage tolerance The linear elastic initiation fracture toughness (K_{IC}) has mostly been used to characterize the flaw tolerance of such composites [1–3]. However, recent studies have shown that aluminium-based metal-matrix composites fracture by stable crack growth and the energy absorbed during crack propagation is a significant fraction of the total energy of fracture [4]. Thus it is essential to study the crack growth resistance in such composites.

2. Model for crack growth

As mentioned above, the flaw tolerance of metal-matrix composites is of critical importance in determining their future use in structural components. A number of methods have been proposed to evaluate the crack growth toughness of materials. Of these, the tearing modulus concept introduced by Paris *et al.* [5] has found wide acceptance. This parameter is a measure of the material's resistance to tearing and an indication of the stability of crack growth. It is defined as

$$T_R = \frac{E}{\sigma_0^2}\left(\frac{dJ}{da}\right) \quad (1)$$

where E is the Young's modulus, σ_0 is the flow stress, and dJ/da is the slope of the J–R curve. The tearing modulus, proportional as it is to dJ/da, is a measure of the strain energy to be provided to the crack tip to enable it to advance by a unit crack length. The nature of the load–displacement curve provides an indication of the value of the tearing modulus. Linear elastic materials exhibit, appropriately, a linear load–displacement trace with crack initiation at maximum load and subsequent unstable crack propagation as depicted in Fig. 1. Elastic–plastic materials, on the other hand, exhibit a non-linear load-displacement curve signifying stable crack growth as shown in Fig. 2. In such materials, crack propagation usually proceeds by a process of void initiation, growth and coalescence leading to a dimpled fracture surface

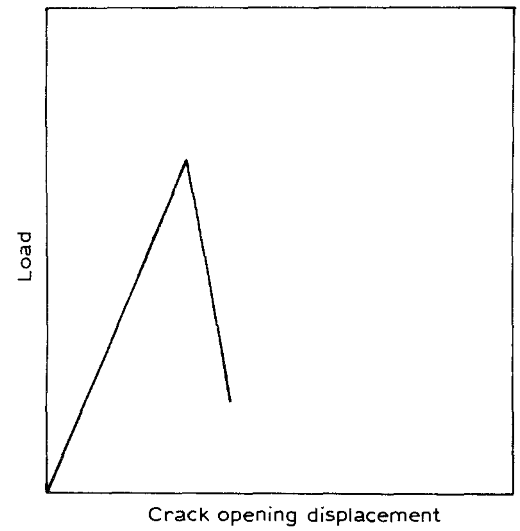

Figure 1 Representative load–displacement plot obtained in a typical fracture toughness test showing unstable crack growth.

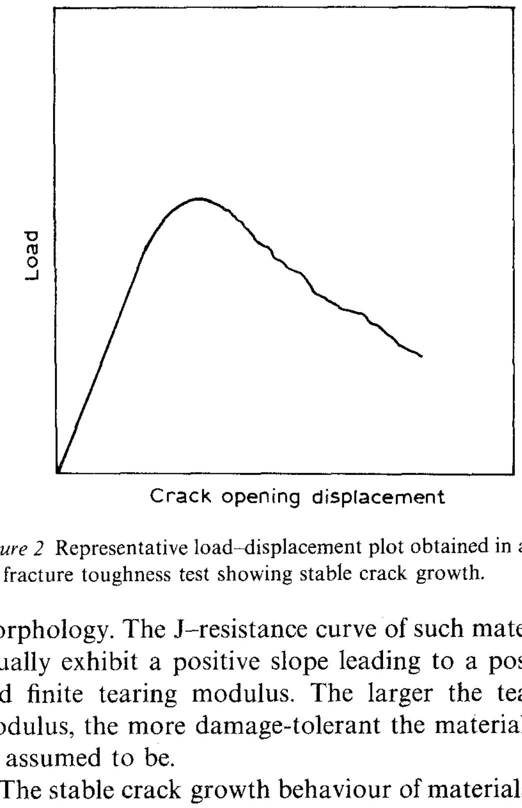

Figure 2 Representative load–displacement plot obtained in a typical fracture toughness test showing stable crack growth.

morphology. The J–resistance curve of such materials usually exhibit a positive slope leading to a positive and finite tearing modulus. The larger the tearing modulus, the more damage-tolerant the material can be assumed to be.

The stable crack growth behaviour of materials has been modelled on a mechanistic basis by a number of researchers [6–11]. The approach originally proposed by Rice and co-workers [10, 11], of the geometrically similar very near crack-tip profile of an extending crack within the asymptotic mode I deformation field of the non-stationary flaw, permits a logical correlation of J_{IC} and the tearing modulus. When evaluated under small-scale yielding conditions, the general expression for T_R simplifies to

$$T_R = T_0 - \frac{\beta}{\alpha_{ssy}} \ln\left(\frac{J}{J_{IC}}\right) \qquad (2)$$

where T_0 is the initial slope of the tearing modulus given by

$$T_0 = \frac{E}{\alpha_{ssy}\sigma_0}\left(\frac{\delta_p}{l_0^*}\right) - \frac{\beta}{\alpha_{ssy}} \ln\left(\frac{esE\,J_{IC}}{l_0^*\sigma_0^2}\right) \qquad (3)$$

and α_{ssy} is the small-scale yielding value of the parameter relating crack tip opening displacement, (CTOD), δ, to J, J is the J-integral and J_{IC} is the J integral at crack initiation, l_0^* is the critical fracture distance, β is a constant = 5.642 for $\nu = 0.3$, δ_p is the crack tip opening displacement for crack propagation, s is the constant relating J and non-stationary crack plastic zone size and e is the natural logarithm base. In particulate-reinforced metal-matrix composites it has been experimentally observed that the tearing modulus does not change with increasing J_{IC} [4] and hence only the initial value of T_R (T_0) will be considered.

To simplify the functional form of the expression relating J_{IC} and T_0, one can utilize the crack growth data of Green and Knott [12, 13] and others [14, 15] and note that the additional CTOD at the advancing crack tip (δ_p) is smaller than the CTOD (δ_i) to cause initiation at the original crack tip. Hence

$$\delta_p = \gamma\delta_i = \frac{\gamma\alpha J_{IC}}{\sigma_0} \qquad (4)$$

where α is the parameter relating δ_i to J_{IC}. Substituting Equation 4 in Equation 3 one obtains, for small-scale yielding:

$$T_0 = \frac{\gamma\alpha}{\alpha_{ssy}}\left(\frac{J_{IC}E}{l_0^*\sigma_0^2}\right) - \frac{\beta}{\alpha_{ssy}} \ln\left[es\left(\frac{J_{IC}E}{l_0^*\sigma_0^2}\right)\right] \qquad (5)$$

In particulate-reinforced metal-matrix composites it has been established that l_0^* is equal to the particle spacing (λ) and that the initiation fracture toughness (J_{IC}) is related to λ [2, 4] in a form originally suggested by Rice and Johnson [16]:

$$J_{IC} = \sigma_0\lambda \qquad (6)$$

Incorporating Equation 6 in Equation 5 and substituting for l_0^*, one obtains

$$T_0 = \frac{\gamma\alpha E}{\alpha_{ssy}\sigma_0} - \frac{\beta}{\alpha_{ssy}} \ln\left[\frac{esE}{\sigma_0}\right] \qquad (7)$$

Using finite-element computations [10] it has been found that $\alpha_{ssy} = 0.581$, $\beta = 5.642$ for $\nu = 0.3$ and $s = 0.12$. The value of α can be taken as 0.5 [17], while γ can be realistically estimated to be 0.25. For an Al–Zn–Cu–Mg matrix reinforced with 15% SiC particulates, the flow stress (σ_0) is 491 MPa and Young's modulus E is 95 GPa [4]. Using the above-mentioned values for the parameters, the tearing modulus is obtained as 1.6 which compares very favourably with the experimentally reported value of 1.84 for this composite [4].

3. Discussion

The primary aim of this paper is to provide criteria for relating the crack growth toughness to initiation toughness as well as microstructural parameters. Such an analysis is essential in predicting the flaw tolerance of composites, especially their ability to undergo stable crack growth for a given combination of stiffness, strength and initiation toughness. However, the nature of the fracture process in composites enables a relationship to be established between initiation fracture toughness and microstructural parameters. Hence, the growth toughness can be directly related to tensile properties. First, let us consider Equation 5 above. The growth resistance should be positive for stable crack growth. In other words

$$\frac{J_{IC}E}{l_0^* \sigma_0^2} > \frac{\beta}{\gamma\alpha} \ln\left[es\left(\frac{J_{IC}E}{l_0^* \sigma_0^2}\right)\right] \quad \text{stable cracking}$$

$$\frac{J_{IC}E}{l_0^* \sigma_0^2} < \frac{\beta}{\gamma\alpha} \ln\left[es\left(\frac{J_{IC}E}{l_0^* \sigma_0^2}\right)\right] \quad \text{Unstable cracking}$$

Thus one can determine whether stable cracking will be observed for a given material. This has been plotted in Fig. 3 where T_0 has been related to $J_{IC}E/l_0^* \sigma_0^2$.

A model for J_{IC} can be derived on the basis of an approach originally developed by Rice and Johnson [16] as outlined earlier. The "characteristic fracture distance" has been shown to be approximately equal to the interparticle spacing λ in a number of studies [2, 4, 13]. Direct evidence for this process was provided by *in situ* deformation experiments in the SEM [18]. In those investigations, tests on both notched and unnotched specimens showed that the fracture proceeded by a microcrack–macrocrack linkage process. The characteristic fracture distance at which this micro–macro crack linkage occurred was of the order of the interparticle spacing. This basis can be used to simplify Equation 5 to a dependence of T_0 on E and σ_0 as is shown in Equation 7. Now, it is possible to predict the stability of crack growth from a knowledge of tensile properties like E and σ_0. Rewriting Equation 7,

$$\frac{E}{\sigma_0} > \frac{\beta}{\gamma\alpha} \ln\left[es\frac{E}{\sigma_0}\right] \quad \text{Stable crack growth}$$

$$\frac{E}{\sigma_0} < \frac{\beta}{\gamma\alpha} \ln\left[es\frac{E}{\sigma_0}\right] \quad \text{Unstable crack growth}$$

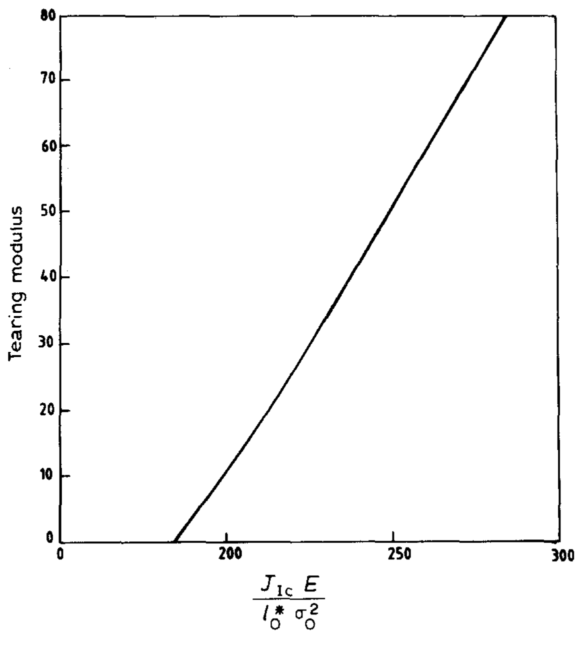

Figure 3 Tearing modulus versus $J_{IC}E/l_0^* \sigma_0^2$.

The aim of developing metal-matrix composites is to attain a maximum in stiffness and strength without compromising unduly on damage tolerance. The above equations can thus provide an indication of the stiffness–strength combinations which can lead to stable crack growth. The stiffness can be varied in metal-matrix composites by changing, for example, the volume fraction of the reinforcement. If the volume fraction is constant, changing the reinforcement size changes the strength without affecting the stiffness. In age-hardenable matrix-based composites, the ageing treatment can be tailored to produce low strength (annealed) or high strength (peak-aged) without a change in the stiffness. Since stiffness gain is usually a predominant factor in favour of metal-matrix composites, the maximum or optimum allowable strength levels can be determined and suitably attained. Since negative values of T_R are not physically tenable, only combinations of E/σ_0 yielding positive T_R values are useful as flaw-tolerant materials; E/σ_0 can then be so chosen that an acceptable value of T_R is attained.

Fig. 4 shows the relationship between T_R and E for three typical strength levels attainable in age-hardenable aluminium alloys. The low strength levels are usually attained by annealing, medium strengths by underageing and high strengths by peak ageing. Fig. 5 shows the effect of σ_0 on T_R for three typical E values corresponding approximately to 5, 15 and 25% reinforcement (like SiC) in the composite. These graphs can also be used in an analogous fashion to arrive at an appropriate combination of E and σ_0 for attaining a given value of T_R.

The relationship between T_R, E and σ_0 discussed above can be extended by incorporating models relating σ_0, E to microstructural parameters. Such an analysis would then lead to a direct dependence of T_R on microstructural parameters. The Young's modulus of composites has been modelled invoking principles like the rule of mixtures, isostress conditions or more complex interactions between the observed variables [19, 20]. One of the more accepted models showing

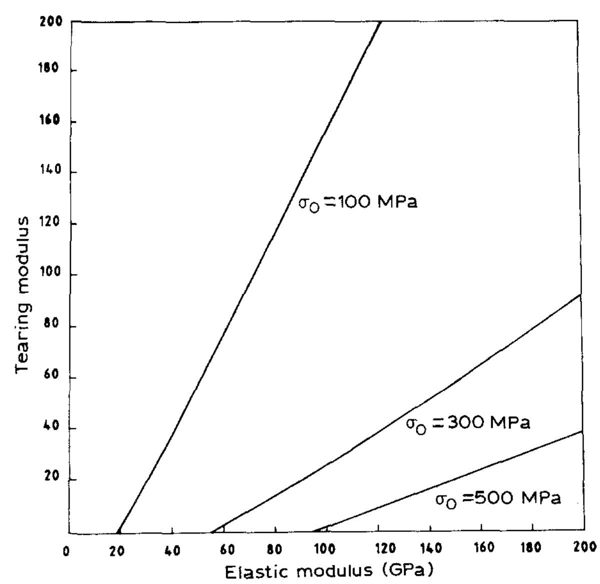

Figure 4. Tearing modulus versus elastic modulus for three typical strength levels.

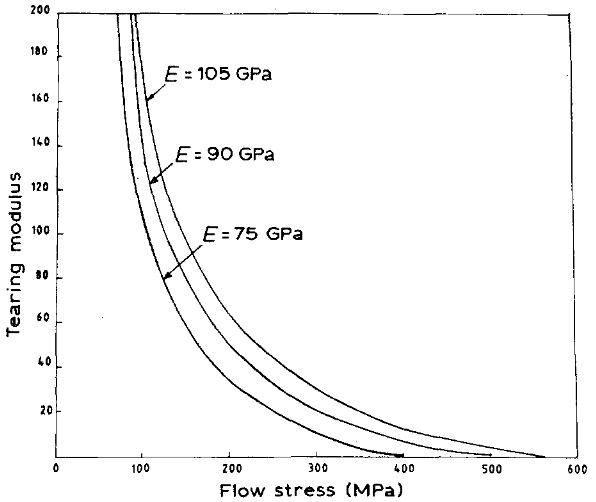

Figure 5 Tearing modulus versus yield strength for three typical elastic modulus values.

good correlations with observed data was proposed originally by Tsai-Halpin and later modified [20]. The modulus of the composite (E_c) can be related to the modulus of the matrix (E_m) through the relation

$$E_c = \frac{E_m(1 + 2qfS)}{(1 - qf)} \quad (8)$$

where E_p is the Young's modulus of reinforcement, E_m is the Young's modulus of matrix, S is the shape factor and

$$q = \left(\frac{E_p}{E_m} - 1\right) \Big/ \left(\frac{E_p}{E_m} + 2S\right)$$

There have been a number of attempts to relate the flow stress of composites to microstructural parameters. Early attempts at such correlations derived their basis from shear-lag theories originally propounded for fibre-reinforced composites. Limitations imposed by the discontinuous nature of the reinforcements, however, limited the scope of these models. Continuum models in general lead to a dependence of flow stress on volume fraction but not on particle size. On the other hand, dislocation-based models lead to a dependence on particle spacing, and thus are influenced by both reinforcement size and volume fraction. The Orowan or cell-size type of model [21, 22] leads to a dependence of the type

$$\sigma_0 = \frac{\alpha\mu\mathbf{b}}{\lambda} \quad (9)$$

where α is the 0.2, μ is the shear modulus, $\mathbf{b}$ is the Burgers vector and λ is the interparticle spacing.

Crystal plasticity models [23, 24] suggest a relation of the form

$$\sigma_0 = \sigma_m + 0.3\mu\left(\frac{\mathbf{b}\varepsilon}{\lambda}\right)^{1/2} \quad (10)$$

where ε is strain.

The form of the model is usually seen to fit the data, though the proportionality constants are larger by an order of magnitude. Such models can be modified by considering shear band formation and pinning due to local incompatibility strains [25]. In any case a λ^{-1} or $\lambda^{-1/2}$ dependence of flow stress is observed.

E can essentially be modified by changing the reinforcement type or volume fraction. Increasing the volume fraction also increases σ_0, thus affecting the E/σ_0 ratio. This can be modified by increasing the particle size which, for a given volume fraction, leads to a small decrease in σ_0. As mentioned earlier, ageing treatments can also be used to modify σ_0 without affecting E. These relationships enable a composite designer to narrow down the variables involved since an optimum of flaw tolerance, strength and stiffness can only be attained for a limited number of combinations of matrix and reinforcement size, type and volume fraction.

4. Conclusions

A model relating the crack growth resistance to the initiation toughness and microstructural parameters of particulate-reinforced metal-matrix composites has been outlined in this paper. On the basis of this model and the value of the initiation toughness and tensile properties, the stability of crack growth can be determined. A critical value of E/σ_0 was found to be necessary to ensure stable crack growth in particulate-reinforced metal-matrix composites. The higher value of E/σ_0, the greater was the material resistance to crack growth.

The value of Young's modulus can be increased, for example, by increasing the volume fraction of the reinforcement, while the flow stress can be optimized by suitable heat treatments. Appropriate variations of reinforcement size for a given volume fraction can alter the strength without sacrificing the Young's modulus, thus enhancing the crack growth resistance. The model can be used in narrowing down the variables involved in designing an optimum composite.

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